COEN 45, MATLAB Programming
Winter Quarter, 2011

Lab assignment #7

Numerical solution of a nonlinear differential equation
Feb. 22, 23, 24

In this lab you will solve this nonlinear differential equation:

$$y\dot{y} - 2t^2 = a$$

where y is a function of time and $\dot{y}$ means dy/dt. A numerical solution, as we have seen in class, means calculating values of y at several time steps starting at $t = 0$ and continuing for a specified duration. You will use MATLAB's `ode45` solver as we have done in class. a is a numerical parameter which you will vary. You will also specify the initial value $y_0 = y(0)$.

1. Following the examples given in class, write a function `ydot_lab7` to compute $\dot{y}$ given y and t, for use by `ode45`. Make a the third input to your function. Vectorize your function even though `ode45` does not need it to be vectorized, so you can use it to calculate the residual as described below.

   ```
   function ydot = ydot_lab7(t,y,a)
   ```

   When you call `ode45` you will pass `a` as the fifth input. The fourth is for specifying options to `ode45` and since we don't usually set any options, provide a null value for this input.

2. When you are satisfied with your function, write a script to use it and get solutions with the following inputs:

   - $y(0) = 10$
   - Time span $0 \le t \le 10$.
   - Separate solutions $y(t)$ for $a = 100, 50, 0, -20$. Plot all four solutions in one figure, using different colors. Use `ylim` to make the y axis run from 0 to 60.
   - Put a legend in the Northwest corner of your figure.

3. Use the y value (for $a = -20$) and your `ydot_lab7` function to show that the equation is really satisfied. Calculate a residual vector $r = y\dot{y} - 2t^2 - a$ and display its maximum absolute value.

4. Using trial and error, find the lowest (most negative) value of a for which the solution converges. You can do these experiments in the command window if you like. (If your solution runs for more than a minute, it is failing to converge and you can kill it with ctrl-c.) Report this number to the nearest tenth. Explain what happened when it failed. (Hint: find the minimum value of y for each trial.)