

COEN 45, MATLAB Programming  
Winter Quarter, 2011

Lab assignment #3  
Flywheel energy storage  
Jan. 25, 26, 27

This assignment provides practice in doing engineering analysis and design calculations, doing units conversion, and plotting. Not all formulas and constants are provided: try Google. For this lab you should review your notes on `fprintf` and element-by-element array operations.

The subject of this assignment is a *flywheel energy storage* device, sometimes called a *mechanical capacitor*. Storage devices are key to development of large-scale wind and solar energy systems because of the intermittent nature of these sources. Kinetic energy is stored in a cylinder that is rotating very fast.

The kinetic energy in a rotating body, as you learned in physics, is  $\frac{1}{2}I\omega^2$  where  $I$  is the mass moment of inertia  $I = \frac{1}{2}Mr^2$  for a solid cylinder and  $\omega$  is its rotary speed in radians per second. If you have a big enough  $I$ , and especially a big enough  $\omega$ , which gets squared, you can potentially store a significant amount of energy. A motor spins up the flywheel when abundant electricity is being generated and a generator converts the energy back to electricity when needed.

Here are some of the practical difficulties:

1. The cylinder must be mounted on bearings that have minimal friction. Otherwise, friction will slow the cylinder down and drain off its stored energy. This is usually solved with magnetic bearings, where the moving parts have no physical contact with the fixed parts.
2. The cylinder must be carefully balanced. Even a little eccentricity could create dynamic forces that could destroy the bearings. If the center of mass is offset from the bearing centerline by an amount  $e$ , a sinusoidal disturbance force is generated with magnitude  $m\omega^2e$ .
3. If the cylinder is rotating very fast, the drag forces that the air exerts on the outer surface could be substantial. This can be solved by mounting the cylinder in an evacuated chamber. The speed of the outer surface is  $V = \omega r$ .
4. High stress can develop in the cylinder. The critical stress is a tensile stress in the tangential direction, given by  $\sigma = \frac{1}{2}\rho\omega^2r^2$ , (expressed in pascals, Pa=N/m<sup>2</sup>, or more conveniently MPa, megapascals).

For more information, look at “flywheel energy storage” on wikipedia.org.

Consider a solid steel cylinder 60 cm in diameter and 1.2 meter long. Its volume is  $V = \pi r^2 \ell$ . Its mass density is  $\rho = 8000 \text{ kg/m}^3$ . Its total mass is  $M = \rho V$  and its mass moment of inertia about its axis is  $I = \frac{1}{2}Mr^2$ . Write a

script called `lab3.m` with the following inputs specified at the top of the script and labelled with units:

- `ss` speed of sound in air, at sea level (m/sec)
- `rad` flywheel radius (m)
- `rho` mass density (kg/m<sup>3</sup>)
- `len` flywheel length (m)
- `ecc` eccentricity: offset from bearing centerline to center of mass (mm)

Consider a solid cylindrical flywheel 1.2 m long and 60 cm in diameter, made of stainless steel.

1. For a rotary speed of 20,000 RPM and eccentricity of 0.01 mm, calculate the following results and use `fprintf` to display them in the command window, formatted nicely, including units:
  - (a) Flywheel length, cm
  - (b) Flywheel diameter, cm
  - (c) Flywheel mass, kg
  - (d) Rotary speed, RPM
  - (e) Energy storage, kw-hrs
  - (f) Outer surface speed, m/sec
  - (g) Outer surface speed, Mach number
  - (h) Tangential stress, MPa
  - (i) Eccentricity, mm
  - (j) Dynamic force amplitude, kN
2. Make the following plots with RPM ranging from 0 to 50,000. Keep the same eccentricity.
  - (a) Kinetic energy in kw-hrs as a function of RPM
  - (b) Tangential stress (in MPa) versus RPM (in a separate figure window).

Label your plot axes, including units, and put your name and student ID in the title. Use `grid`.

3. Perform a design tradeoff using 60,000 RPM. Assume the total cost of a flywheel is given by the empirical formula

$$C = 100 + 80d^{1.4} + 0.5M^2$$

where  $d$  is the diameter in cm and  $M$  is the mass in kg. Plot cost per kw-hr versus diameter. Note on your plot the approximate diameter that gives minimum cost per kw-hr.

As you can see you have a lot of units conversions to make. All calculations should be done in SI units: m, kg, sec, N. Submit printouts of your script and both plots.